

Pythagorean Figured Numbers:
Oblong, Square, Triangular, etc.
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Here we go on Oblong numbers...

The problem is that they are much easier to explain with pictures, as the Pythagoreans did, than with words, to which ASCII is suited. But I'll try to do pictures.

The easiest to understand are the Square numbers; these are ones that can be arranged in a square:

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*      **      ***      ****      *****      ...
      **      ***      ****      *****
            ***      ****      *****
                      ****      *****
                                *****

1       4       9       16       25       ...

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Well, they don't look quite square on my terminal; maybe they do on yours! Mathematicians still call these numbers squares: 1 squared = 1, 2 squared = 4, 3 squared = 9, etc. There are lots of interesting things about them, especially from a Pythagorean standpoint, but let's go on to the Oblongs.

An Oblong number can be arranged in a rectangle whose width and height differ by one unit; for simplicity I'll make them wider than tall. So whereas the Square numbers have sizes 1 X 1, 2 X 2, 3 X 3, 4 X 4, etc. the Oblong numbers have sizes 1 X 2, 2 X 3, 3 X 4, etc. So they look like this:

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                                1009*3027
                                1010*3030
                                1011*3033
                                1012*3036
                                1013*3039
                                1014*3042
                                1015*3045
                                1016*3048
                                1017*3051
                                1018*3054
                                1019*3057
                                1020*3060
                                1021*3063
                                1022*3066
                                1023*3069
                                1024*3072
                                1025*3075
                                1026*3078
                                1027*3081
                                1028*3084
                                1029*3087
                                1030*3090
                                1031*3093
                                1032*3096
                                1033*3099
                                1034*3102
                                1035*3105
                                1036*3108
                                1037*3111
                                1038*3114
                                1039*3117
                                1040*3120
                                1041*3123
                                1042*3126
                                1043*3129
                                1044*3132
                                1045*3135
                                1046*3138
                                1047*3141
                                1048*3144
                                1049*3147
                                1050*3150
                                1051*3153
                                1052*3156
                                1053*3159
                                1054*3162
                                1055*3165
                                1056*3168
                                1057*3171
                                1058*3174
                                1059*3177
                                1060*3180
                                1061*3183
                                1062*3186
                                1063*3189
                                1064*3192
                                1065*3195
                                1066*3198
                                1067*3201
                                1068*3204
                                1069*3207
                                1070*3210
                                1071*3213
                                1072*3216
                                1073*3219
                                1074*3222
                                1075*3225
                                1076*3228
                                1077*3231
                                1078*3234
                                1079*3237
                                1080*3240
                                1081*3243
                                1082*3246
                                1083*3249
                                1084*3252
                                1085*3255
                                1086*3258
                                1087*3261
                                1088*3264
                                1089*3267
                                1090*3270
                                1091*3273
                                1092*3276
                                1093*3279
                                1094*3282
                                1095*3285
                                1096*3288
                                1097*3291
                                1098*3294
                                1099*3297
                                1100*3300
                                1101*3303
                                1102*3306
                                1103*3309

```

Notice that twice these are the Oblongs, $2 \times 1 = 2$, $2 \times 3 = 6$, $2 \times 6 = 12$, etc. You can also see this geometrically; take the Oblong number 12 and see how it can be divided into twice the triangular 6:

```
* * * / *
* * / * *
* / * * *
```

So there's a taste of Pythagorean number theory. The triangular number Ten, the Decad, is very important, since it is the Tetraktys:

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  *
 * *
* * *
* * * *
```

"The Power, Efficacy and Essence of Number is seen in the Decad; It is great, It realizes all its purposes, and It is the Cause of all Effects. The Power of the Decad is the Principle and Guide of all Life, Divine, Celestial, or Human into which It is insinuated; without It everything is indefinite, obscure, and furtive." (Stobaeus 1.3.8)

Pythagoreans take an oath by saying,

"I swear by the One who hath bestowed the Tetraktys to the coming generations, source of Eternal Nature, into our Souls."

>From a musical standpoint its ratios (1:2, 2:3, 3:4) generate the octave, fifth and fourth.

By the way, though Triads are very important in Arithmetical Theology, the Tetraktys shows us that Dyads and Tetrads are also important, as several other posters have mentioned.

I hope that's of some use to those not familiar with Pythagorean number theory, and not to boring for those who are!

Hugieia,
John Opsopaus

P.S. "Hugieia" (well-being, soundness in body and mind) is a Pythagorean salutation, also used as a talisman when written in the points of a pentagram so: U/G/I/EI/A (in Greek letters, of course). -- JO

THEOLOGIA ARITHMETICA TRIADUM

Arithmetical Theology of triads

John Opsopaus

Here are some observations, mostly Pythagorean but also Alchemical, about triads, both theological and musical.

I. TRIADIC GENESIS

A. The Monad

The first principle is the Monad or Unit. It is *prima materia* (first matter), an undifferentiated unity, an unstructured wholeness, the principle of stability. The Monad corresponds to the first stage of many cosmologies, the Primeval Waters, the Confused Mass (*Massa confusa*), Tiamat, etc.

B. The Dyad

1. Undetermined Dyad

The second principle is the Undetermined Dyad (*aoristos duas*), the principle of change, separation, differentiation, conjunction and mediation. The Indefinite Dyad is an active/energetic principle, as opposed to the Monad, which is passive/material.

2. The Dyad Itself

The Undetermined Dyad becomes determinate by operating on the Monad. As the principle of separation (*Separatio*) it splits the Monad, so that Chaos (gap) opens in it. In this way the Indefinite Dyad creates the Dyad Itself (*auto to esti duas*), an actual separation into opposites (*Separatio oppositorum*). It creates a polarity between male/female, form/matter, stability/change, etc. They are the passive forms of the alchemical Sulphur and Mercury. The Dyad corresponds to the second cosmological stage, in which some active agent (e.g. Marduk, Shu, "Destructive Strife" -- *oloios neikos*, etc.) opens a gap between Earth (Gaia, Keb) and Sky (Ouranos, Nut); it's the splitting of the Cosmic Egg.

C. The Triad

The Undetermined Dyad operates again, this time as a principle of mediation, to create a Mean between the Extremes of the Dyad, thus producing the Triad, which is a *Harmonia* (joining), a structured unity, the *Rebis* (Two-thing), a synthesis of the opposites (*Coniunctio oppositorum*) of the Dyad. This is the Alchemical Wedding, in which the alchemical Mercury attracts alchemical Sulphur towards itself so that they can be joined by means of the alchemical Salt; the result is the Living Gold or Philosophers' Stone. The Triad corresponds to the third cosmological stage in which some active agent (e.g. Eros) causes Earth and Sky to mate and beget the Gods.

These are some aspects of the Polarity and its Harmonia in the Triad:

Male	Female	Androgynous
Form	Matter	Formed Matter
Stability	Change	Lawful Change
Fixed	Volatile	Fixed-Volatile
Unification	Separation	Harmonia
Emission	Reception	Reciprocation
Sulphur	Mercury	Salt

D. Higher Orders

The Undetermined Dyad can operate, through Separatio and Coniunctio, to form Tetrads, Pentads, and so forth in a cyclic evolution, but the first three are the most important.

II. THE SACRED FAMILY (Synchronic Triad)

The Triad can manifest in the Godhead in many different ways, but one of the most common is as three archetypal genders, masculine, feminine and neuter (or androgynous). (Needless to say, they are only loosely correlated with biological sex.) Most commonly the Triad appears as a Sacred Family of three Gods, a Father, a Mother and Child, whose characteristics often derive from those of the Triad.

A. Structure

1. Father

The Father is the form-providing and matter-forming element of generation, the seed, and represents stability and rigidity, conscious soul (anima), emission, creation, and isolation. His element is Air (or Air and Fire) and His planet is the Sun.

2. Mother

The Mother is the matter-providing and form-receiving element of generation, the womb, and represents change and fluidity, vital spirit (spiritus), reception, redemption, and the possibility of relation. Her element is Water and Her planet is the Moon.

3. Child

The Child is the formed matter resulting from generation (through the mediation of Harmonia, the Mean), the offspring, and represents controlled change, equal exchange, the spirit uniting the soul with the body, the actualization of relation, resurrection, and balance in all the oppositions of the Father and Mother. Its element is Earth (and perhaps Fire) and Its planet is Mercury.

B. Examples

1. Capitoline Trinity: Jupiter-Juno-Minerva

Note that Minerva (like Athena) is one of the more androgynous Goddesses, since Her personality integrates both archetypally masculine and archetypally feminine characteristics. (Of course, in the Greek version Juno is not Her Mother, though Juno does provoke Jupiter into begetting Her.)

2. Egyptian Trinity: Osiris-Isis-Horus

3. Wagnerian: Wotan-Erda-Bruenhilde

Bruenhilde is also androgynous, exhibiting archetypally masculine characteristics in Walkure, transforming in Siegfried, and becoming archetypally feminine in Goetterdaemmerung.

4. Christian: Yahweh-Mary-Jesus

5. Chthonic Trinity: Hades-Demeter-Persephone (Hecate)

This Trinity requires some explanation, since Hades-Demeter-Persephone do not fit the Father-Mother-Child pattern. However, Demeter is Persephone's Mother, and Hades is Persephone's Uncle (since Zeus is Her Father). Furthermore, since Hades represents the World of the Dead and Demeter the World of the Living, we see that Persephone becomes the Harmonia that joins the two, and reconciles the two adults; the two realms are "related by marriage." Hecate plays an interesting role in the myth, for She is both an Underworld Goddess and a Moon Goddess and so transcends the divide. She seems to fill the role of the Undetermined Dyad, since She is the mediator, first when She tells Demeter where Persphone has been taken, and second when She becomes Persphone's guardian during Her time in the Underworld.

III. THE SACRED LIFECYCLE (Diachronic Triad)

Whereas the Sacred Family is a static relationship within a Triad of Gods, the Sacred Lifecycle is a Triad in the life of an individual, which manifests as a Triple God or Goddess; I'll call the stages Virgin, Parent and Sage. In many cases these Triads are cyclic.

A. Structure

1. Virgin

The Monad corresponds to the first stage of life, the Virgin, and is characterized by self-centeredness, individuality, lack of discrimination and ignorance of norms.

The Feminine Virgin is the archetypal Maiden, and the Masculine is perhaps the archetypal Hero. The characteristic activity of the Hero is protection; that of the Maiden is perhaps inspiration.

2. Parent

The Dyad corresponds to the second stage of life, the Parent, and is characterized by other-directedness (i.e., caring, supporting, self-sacrificing), excessive competition or cooperation (interaction with others), inflexible discrimination, and rigid adherence to norms.

The Feminine Parent is the archetypal Mother, whose characteristic function is nurturing; the Masculine Parent is the archetypal Father, whose characteristic function is providing.

3. Sage

The Triad corresponds to the third stage of life, which integrates the preceding, and is characterized by balanced devotion to self and others, balanced competition/cooperation, flexible discrimination, and transcendence of rules. It is the synthesis of the thesis and antithesis of the first two stages.

The Masculine Sage is the Wiseman, whose characteristic function is "professing" (philosophers, priests, statesmen); the Feminine Sage is the Crone, whose characteristic function is perhaps healing (counsellors, witches, healers).

B. Example

1. Olympian Cycle: Dionysus(=Son of Zeus)-Zeus-Kronos

2. Chthonic Cycle: Persephone-Hades/Demeter-Hecate

Persephone is the archetypal Maiden engaged in self-centered enjoyment, when She unwittingly inspires Hades to abduct Her. Demeter reacts as the archetypal Mother. Hades of course is not Persephone's Father, but He does at least obey the norms of the time by securing Her Father's consent and making Persephone His legitimate wife and queen; in this sense He fits the Parent archetype. Hecate is a archetypal Crone, working for a practical resolution of the situation.

IV. GENESIS OF MUSICAL HARMONIA

In this section I'll show how the same triadic structure produces the Harmonia (tuning) of musical scales.

A. Triadic Genesis

1. Unison

The Monad is the Perfect Unison, two sounds of the same pitch (C-C). The Perfect Unison is the most consonant interval.

2. Octave

The Dyad is the Octave, since a string divided in half will sound an octave above the undivided string (C-c). Further, if a string is divided in the ratio 1:2, its parts will sound an Octave apart. The Octave is the next most consonant interval after the Perfect Unison.

3. Perfect Fifth

The Triad is the Perfect Fifth, since if a string is divided in the ratio 2:3, the parts will sound at the interval of a Perfect Fifth (C-G), which is the next most consonant interval after the Octave.

B. Tetraktys

The Tetraktys, a triangle of units in rows of size 1:2:3:4, was a central mystery of the Pythagoreans:

"What is the Oracle at Delphi?"

"The Tetraktys, the very thing that is the Harmonia of the Sirens."
(Iamb., Vit. Pyth. 18)

The Tetraktys embodies the first three consonant intervals after Perfect Unison: the Octave (1:2), the Perfect Fifth (2:3), and the Perfect Fourth (3:4), e.g. C-F.

Because the interval between the Perfect Fourth and the Perfect Fifth is a whole tone, it provides the basic unit for constructing a scale. Ancient lyres were tuned as follows: the tonic (hypate), the octave (nete), the fifth (trite or paramese), the fourth (mese). The tuning of the remaining strings would determine the Harmonia (tuning or mode) of the lyre. (The earliest music was limited to just the four tones, "the Harmonia of the Sirens.")

C. Oblong Numbers

The Pythagoreans classified numbers as Triangular (1, 3, 6, 10,...), Square (1, 4, 9, 16,...), Oblong (2, 6, 12, 20,...), etc., depending on the shapes into which they could be arranged. The Oblong numbers are those that can be arranged in a rectangle one unit wider than it is high; each is twice a Triangular number. The Oblongs have sides in the ratios 1:2, 2:3, 3:4, 4:5, 5:6,.... These give the intervals in decreasing order of consonance: Octave, Perfect Fifth, Perfect Fourth, Major Third (4:5), Minor Third (5:6), etc. The Tonic, Third and Fifth are the basic components of Triadic Harmony.

D. The Circle of Fifths

The basic process of the Triadic Genesis can be repeated to generate all the pitches of the chromatic scale:

- i. The Monad: Choose a pitch.
- ii. The Dyad: Double the pitch and so determine the Octave.
- iii. The Triad: Take the (Arithmetic) Mean of the Octave to get the Perfect Fifth.
- iv. Now take this new pitch as the Monad and repeat the above steps.

This will produce that well-known "Circle of Fifths" that will eventually generate all the pitches of the chromatic scale and return to the starting point (after 12 cycles):

C	G	D	A	E
B	G ^b	D ^b	A ^b	E ^b
B ^b	F	c		

(This will require slight adjustment to correspond to modern equal-temperment

tuning, which replaces the Pythagorean exact ratio $3/2 = 1.5$ with the approximation $2^{7/12} = 1.49831.$)

*** finis ***

THE PENTAGRAM AND THE ELEMENTS
John Opsopaus
November 1993

INTRODUCTION

In many contemporary Wiccan Traditions, as well as the Golden Dawn and other schools of ceremonial magic, the elements are associated with the points of the Pentagram in the following way:

Spirit
Air Water
Earth Fire

I have reason to doubt it, at least from the perspective of the apparent Babylonian origins of the Pentagram as well as Babylonian, Egyptian and Greek astrology, and, more recently, in Alchemy.

The GD/Wiccan pattern seems to be traceable back to Levi but no further. While I agree with the philosophy, "If it works for you, use it," I think that structures of ideas tend to work better to the extent that they have a high degree of internal coherence -- thus the importance of understanding the "inner logic" of the Pentagram/Element connection. While the GD/Wiccan system has its logic, I suspect it's a comparatively recent invention, and some of the older systems seem more comprehensive, to my mind, anyway. I cannot offer a definite alternative at this time, but I can show some of the pieces I'm trying to fit together. If anyone can help, I would certainly appreciate it.

THE PLANETS THE ELEMENTS AND THE DIRECTIONS

There are (at least) three interrelated systems (evidence cited later) connected with the Pentagram: (1) The Five Planets, (2) the Four Elements + Quintessence, and (3) the Four Directions. Henceforth,

Four Directions: NESW

Four Elements: EAFW + Q

Five Planets: (K = Saturnus [Kronos], J = Juppiter, M = Mars,

H = Mercurius [Hermes], V = Venus), plus S = Sol, L = Luna.

A further complication is that we can go around the pentagram two ways: (1) circularly, around the outside, and (2) linearly, tracing the "three interlaced triangles." If we number the points in a circular circuit 12345, then the linear circuit (in the same direction) visits them 13524; this is important when the p.g. represents the stages of a process.

Working backward in time:

An alchemical image of "Man as Microcosm" gives one of the only direct associations of elements with the pentagram:

Q
A F
W E

It is reproduced in Godwin (Alchemical Mandala, 11), but I don't know its original source. Another alchemical diagram (Godwin, 20; original source unknown) gives the following:

F A
Q
E W

There is ancient support (see below) for the center of the square being equivalent to the apex of the pentagram, so this would correspond to:

Q
F A
E W

which is a logical arrangement (given the square of opposition of the elements), but not the usual GD/Wiccan one; it's also the mirror image of the first p.g. given above.

Other alchemical sources (e.g. Jamsthaler 1625) arrange the planets pentagonally, typically to represent stages in the alchemical procedure:

S L
H
M J
V K

The Moon (L) indicates the passive ascending process and the Sun (S) indicates the active descending process (Burckhardt, Alch. 193). The five lesser planets (HMJVK) form the pentagram, with Mercury (H) at the apex; a position

which has much ancient support (see below). Some alchemists (e.g. Valentine, 1659) invert both processes:

```

      H
V      K
M      J

```

Burckhardt shows that another arrangement (Boehme 1628, Stolcius 1624), which brings Venus and Jupiter together, aligns corresponding steps of the ascent and descent:

```

      S      L
      H
M      V
      K      J

```

The latter arrangement also agrees with the standard association of the planets with the hand known from palmistry (the p.g. being 1=J, 2=K, 4=H, palm=M, thumb=V), though it unfortunately reverses S (3) and L (heel). Despite the confusion, all seem to agree in attaching special significance to Mercury and in putting Jupiter/Saturn on one side and Venus/Mars on the other.

The pentagrams in Agrippa (De occulta phil., 1533) are associated with the planets; these are what we find (Lib. II):

```

      M              L S V
V S J              J      (the latter not really a p.g., but easily
      L              M      connected with it via the human figure)
H K              H K

```

He also shows the planetary assignment of palmistry, which seems unrelated to these others, but matches better some alchemical associations (see above). The planetary associations can be converted to elementary associations by means of the Scala Quinarum (also in Lib. II), which gives K/W, J/A, M/F, V/E, H/M (mixtum), and hence for the first pentagram:

```

      F
E      A
M      W

```

However, I don't have great confidence in his planet/element associations (though if F and M were switched I'd buy it).

In Lib. III we find the HUGIEIA (Soundness) Pentagram of the Pythagoreans, with U/G/I/EI/A clockwise from the top over an inverted pentagram. However, this doesn't help us with elemental associations, though it falls in a tradition of writing magic words around pentagrams (e.g. te/tra/gram/ma/ton) and other pentacles, which dates back to at least the 4th or 5th cent. BCE (Jewish jar handles from Palestine). (The pentagram itself goes back to Mesopotamia -- early 3rd mill. BCE; see below.)

This is what I've found so far on the Babylonian, Egyptian and Greek evidence on the relation of the Pentagram and the Elements. It should be considered work in progress, and any suggestions or other help would be greatly appreciated.

BABYLONIA:

The Pentagonagram appears as a sign (UB) in the earliest form of Sumerian pictographic writing (c. 3000 BCE). Although such pictographs do not have a unique meaning, the general sense seems to be "heavenly body." By the cuneiform period (say, after 2600 BCE) the pentagram means "region," "heavenly quarter" or "direction" (Forward Backward Left Right) and is generally used with the number 4. It is conjectured that early Babylonians added a fifth direction (Up) because they believed Heaven ruled the Four Quarters; de Vogel makes an analogy with the Jewish Prayer of the Night in which Schekina rules over the Four Archangels (Michael, Raphael, Gabriel, Uriel). In Babylon the Four Directions were ruled, according to de Vogel, by the planets Jupiter, Mercury, Mars and Saturn, but he doesn't say which ruled which; Venus, the fifth planet, corresponded to the Queen of Heaven (Ishtar). (De Vogel, Pyth., App. A)

Although I've been unable to find any evidence for a Babylonian theory of the Elements, there are suggestive indications. For example in the Babylonian Creation Myth "Enuma Elish" (before 1000 BCE) and elsewhere Marduk assigns The Four Worlds as follows: Luminous Heaven: Anu, Sky: Enlil (Lord Storm), Earth: Marduk, Watery Abyss: Ea. Thus we have rather clear elemental associations, Anu: fire, Enlil: Air, Marduk: Earth, Ea: Water. Unfortunately only one of these is a planet: Marduk = Jupiter. (King, Enuma Elish, xlix, lxxxiv; Kramer, Sum. Myth., 41; Perry, Lord Four Quart., 75-80)

In anticipation of considering the Greek system I'll observe that there may be a basic incompatibility between the Babylonian and Greek conceptions of the elements. In Babylonia the typical arrangement is (from top down): Fire, Air, Earth, Water, since the Abyss is under the Earth. In Greece (at least as early as Heraclitus, c.500 BCE) the order (from top down) is: Fire, Air, Water, Earth, since this is the order of increasing density which forms the basis of Plato's Cycle, which is fundamental to alchemy. In both systems, Spirit (Aether etc.) is above the Four Mundane Elements, but the arrangement of the latter may ultimately depend on whether you prefer the Babylonian or Greek scheme.

The preceding gives one correlation between Elements and Planets but, unfortunately, I've been unable to establish a credible relationship between the other Planets and Directions in Babylonia. The Seven Planets correspond to Gods: Sin/Moon, Shamash/Sun, Marduk/Jupiter, Ishtar/Venus, Ninurta/Saturn, Nergal/Mars, Nebo/Mercury (in their canonical order; yes, the Moon originally had priority over the Sun). Further, the division of Heaven and Earth into Four Quarters was a central part of Babylonian cosmology. Interestingly, when they assigned things (such as months or days) to the Directions, they did so in the order SEWN, which is reminiscent of the Cabalistic Tree of Life. If we take the Gods Ruling the Four Elemental Worlds and assign them to the directions in their usual order (Anu, Enlil, Ea, Marduk), then we get:

N: earth/Marduk
W: water/Ea E: air/Enlil
S: fire/Anu

This is certainly a reasonable and familiar arrangement for the Elements, but since it involves only one Planet, its connection with the Pentagonagram is tenuous.

Unfortunately, I've been unable to find other direct associations between the Planets and the Directions, and indirect assignments (via lists of Gods, or the Months and their Gods) have produced dubious results; that is, none associates one Planet with each Direction. If we assign the planets in their constant Babylonian order (L/Luna, S/Sol, J/Jupiter, V/Venus, K/Saturn, M/Mars, H/Mercury) to the directions SEWN, omitting Venus (which is Up), then we get the following Directional arrangement for the Planets:

	K	
J		H
	M	

which is reasonable, but has little to support it (and disagrees in its placement of Jupiter/Marduk, though here He is in the context of a different rank of Gods).

(see Jastrow, Rel. Bab. Asyr., Ch. 22)

Although the Babylonian system, with the special significance it attaches to Venus, does not agree with the Egyptian or Greek systems, which distinguish Mercury, we can perhaps take as a general principal that the distinguished planet corresponds to Up or Center, and the remaining four to the Quarters.

EGYPTIAN:

I have found no evidence of a specific theory of Four Elements in Egypt, although, as is common in many cultures, there is informal recognition of parts of the Cosmos that correspond, more or less, to the elements. On the other hand, Egyptian cosmology is based on a system of Four Directions, so there is some hope of making the connection between the Four Planets and the Directions. However, I've not yet been able to do so. What I've found is the following:

In Manetho's "Epitome of Physical Doctrine" (fr. 83) we have a system of correspondences that explicitly associates the Five Elements with Five Gods:

Spirit: Zeus [= Amon?]
Fire: Hephaestus [= Phah?]
Air: Athena [= Neith?]
Water: Ocean (Nile) [= Nile]
Earth: Demeter [= Geb?]

In addition, Moon and Sun are associated with Isis and Osiris. Unfortunately, there is no connection given between the Directions and either the Gods or Elements.

Also suggestive is the Ogdoad of Hermopolis, which comprises four male-female couples. Two of the four are clearly associated with Elements, but I know of no directional associations:

Nun & Naunet: Abyss (water)
Huh & Hauket: Expansiveness (fire?)
Kuk & Kauket: Darkness (earth?)
Amun & Amaunet: Hidden (air)

These Eight are considered the Souls of Thoth = Hermes = Mercury, which is one instance of the Egyptians, like the Greeks, making Mercury the distinguished planet/God (rather than Venus, like the Babylonians).

The various Enneads (Heliopolis, Greater, Lesser) are also suggestive (1 + 4 pairs), but I've been unable to find clear elemental or directional associations.

Though I haven't been able to find a conclusive association between the Pentagram and the Elements in any Ancient Tradition, there is quite a lot of suggestive material, and I can do a reasonable (re)construction. After summarizing the conclusions from the Babylonian and Egyptian material, I'll turn to the Greek.

SUMMARY:

The Pentagram originated as a five pointed star in the earliest pictographic writing in Babylonia, with a meaning something like "a quarter of the heavens." Scholars have conjectured that the five points correspond to Four Directions (originally Front/Back/Right/Left) and Up; they further conjecture that Up was associated with Ishtar/Venus, the Queen of Heaven, and that the other Four Directions were associated with the other four Minor Planets (Mercury, Mars, Jupiter, Saturn).

I conjecture that this basic 4+1 pattern was imported into Egypt and Greece (the latter via Pythagoras), but that they both made Thoth/Hermes/Mercury the distinguished planet (rather than Venus). So then, the problem reduces to associating these Planets or Gods with the Directions, associating the Directions with the Pentagram, and associating the Elements with the Planets or Gods.

GREEK EVIDENCE:

First I'll deal with the Four Elements, and then consider the Pentagram and the Five Elements. Some of the preliminaries will be very familiar to everyone on this list.

My major source is Ptolemy's Tetrabiblos (I.4-8, 10, 17, II.3) which gives a explanation of the associations. However, I think Aristotle's discussion (e.g. De caelo 268b11-296a32; De gen. & corr. 329a24-331a6) of the Four Elements and Four Qualities is much more profound, and is really necessary for seeing the significance of this analysis. See also Plato (Tim. 53c-64a).

Ptolemy's (like Aristotle's) is based more on the Four Qualities than the Four Elements. The Four Qualities comprise two Dualities: Hot/Cold (active) and Dry/Wet (passive), usually displayed in a Square of Opposition, thus:

```
      C
    W   D
      H
```

(NB: This does NOT produce the familiar Wiccan/Golden Dawn (for lack of a better name) arrangement of the Elements, but more on that later.)

The Four Qualities also fall into a Natural Order, which might be called the Organic Order: Wet, Hot, Dry, Cold (the preceding circle counterclockwise). I won't repeat Ptolemy's explanations, but here are some of the correspondences he derives:

Quality	Age	Season	Moon	Winds

Wet	Child	Spring	1st Q.	W
Hot	Youth	Summer	2nd Q.	S
Dry	Middleage	Autumn	3rd Q.	E
Cold	Old/death	Winter	4th Q.	N

Note that this makes the Cold/Wet transition of the Cycle a key Transition Point: (1) it represents the Dark of the Moon, (2) the traditional start of the year (Vernal Eq.), and (3) reincarnation (the passage from death back into birth). Note that the Directional associations are made via the Winds (although there are other Directional assignments based on Government of the Trines). I've worried about the Directions, since they are crucial, but have finally come to accept Ptolemy's assignment.

It's also important to keep in mind another Order, which I'll call the Physical Order and goes in increasing or decreasing Density (or Subtlety): Earth, Water, Air, Fire; thus Earth and Fire are the Extremes (Absolute Heavy/Light) and Water and Air are the Means (Relative Heavy/Light). The corresponding order for the Qualities is: Dry/Cold/Wet/Hot/Dry. The Physical Order is also a Physical Cycle, for example in the Circulation of the Elements in Alchemy. Thus, in addition to the Organic Transition Point between Cold/Wet = Water (Vernal Eq.), there is also a Physical Transition Point at Dry (Samhain).

(In most Greek philosophy -- certainly Aristotle's -- the Physical Order is also the Cosmological Order: the arrangement of the Levels (Spheres or Planes) of the World; note however, that the Cosmological order in Babylonia and many other places is Water, Earth, Air, Fire, since the Watery Abyss lies

under the Earth. This is the important arrangement for Shamanic Traveling, and is expressed more accurately as Abyssal Water, Earth, Air, Celestial Fire.)

Ptolemy's assignment of qualities to the Five Minor Planets is more complex, but when all is said and done, it works out fairly naturally.

Planet	Qualities	Element	Dominant Element
Mercury	Dry/Wet alternating	*	*
Venus	Wet(Hot)	~Air	Water
Mars	Dry, Hot	Fire	Fire
Jupiter	Hot(Wet)	~Air	Air
Saturn	Cold(Dry)	~Earth	Earth

Note that Mercury, unique among the Planets, is Duplex in displaying an alternation of opposed Qualities. The "Element" column shows the Element corresponding to the pair of listed Qualities, which does not work well. Therefore, in the "Dominant Element" column, I've associated the Element based on a direct Element/Quality association (Water/Wet, Fire/Dry, Air/Hot, Earth/Cold). Aristotle does something similar, though he takes the other Quality out of each pair associated with an Element (Water/Cold, Fire/Hot, Air/Wet, Earth/Dry). The issue is simply whether the Element is represented by its clockwise or counterclockwise neighbor in the Square of Opposition:

Water	Earth
Cold	
Wet	Dry
Hot	
Air	Fire

(NB: It's traditional to place the Elements at the corners of the Square of Opposition and the Qualities on the edges. However, this is not the best arrangement for, as Aristotle says, the Qualities are absolute and primary, whereas the Elements are always relative mixtures and secondary. Therefore the Qualities should be placed at the corners; then the edges can represent various proportions of the Qualities constituting an Element, e.g., Earth that is more Dry or more Cold.)

The associations I've taken seem to work better overall, but the point is arguable. Also it's important to keep in mind that in associating an Element with a single Quality we are making an approximation; the Elements actually belong *between* the Qualities, and thus *between* the Directional Quarters, Seasons, Moon Phases, etc. Be that as it may, when these Elemental associations are combined with the Qualities, we get:

Quality	Age	Season	Moon	Dir.	Element	Planet/God
Wet	Child	Spring	1st Q.	W	Water	Venus (Born of Seafoam)
Hot	Youth	Summer	2nd Q.	S	Air	Jupiter (Sky God)
Dry	Middleage	Autumn	3rd Q.	E	Fire	Mars (Burning Red)
Cold	Old	Winter	4th Q.	N	Earth	Saturn (Earth/Vege. God)
--	Death	--	--	Up	Spirit	Mercury (Guide of Souls)

The relation of the Five Elements and Four Qualities is neatly displayed by the Wheel of Heaven: Draw an equal-armed cross in a circle. The cross arms represent the Four Qualities (or the Four Elements). The center of the Cross and Circle represents the featureless Substratum, the Prima Materia, wherein all qualitative difference, all duality disappears (it is, if you like, the "zeroth" substance). The bounding Circle represents the Quintessence, the Aither, the "Sphere of the All" (Aetius), which Plato associated with the Dodecahedron (a mystic figure, made of Pentagons, and like the Pentagram, used as a symbol of recognition by the Pythagoreans). As Plato says, "The God made the Dodecahedron the Image of the Universe, as being the nearest to the Sphere." It is the Spirit governing the Four Mundane Elements.

According to the Timaeus, the Four Elements are the offspring of Concrete Matter (the Center) and Abstract Form (the Circle).

Now that we have the cyclic arrangement of the Four Elements, we can consider how they are assigned to the Pentagram.

* * *

I've previously shown the correspondences between Elements, Qualities, Directions, Seasons, Ages and Planets that results from Ptolemy's analysis. Further, I'm operating on the hypothesis that the five points of the Pentagram correspond to the Four Directions and Up. Therefore, think of a five-rayed star as a perspective picture, with the lower cross representing the Four Directions and the vertical ray representing Up. Depending on the direction we're facing, there are four possible arrangements:

*	*	*	*
N E	E S	S W	W N
W S	N W	E N	S E

I will argue that the first arrangement is the correct one. To this end, I'll use the Directional Correspondences to label the Pentagram with Elements, Planets, etc. For improved readability I'll use:

E=Earth, W=Water, A=Air, F=Fire, S=Spirit
H=Mercury, V=Venus, M=Mars, J=Jupiter, S=Saturn

Elements Planets

S	H
E F	S M
W A	V J

To see what's going on more clearly, you might want to draw a Pentagram with a circumscribed Pentagon. It will be helpful later if you draw the Pentagram and Pentagon in contrasting colors. The the lower trapezoid of the figure is then a (distorted) Square of Opposition representing the Four Elements EWAFF, Directions NWSE, and Planets SVJM.

First observe that this Pentagram embodies the Physical Order of the Elements; if we stay on the Mundane level we have EWAFF, and we can make it a cycle counterclockwise (as in the Alchemical Circulation) by returning from F to E across the horizontal beam of the Pentagram. Likewise, the Pentagram includes the Extended Physical Order (the Metaphysical Order), which includes the ascent to Spirit: EWAFS by a counterclockwise circuit. Of course we get the descent SFAWE by going clockwise.

Also, the Organic Order WAFE = spring/summer/fall/winter is still embodied in the lower trapezoid, though the Organic Transition Point (Water) is the left leg of the Pentagram, an undistinguished position.

Notice, too, how Spirit, with its four descending lines, surrounds the Cross of Opposition with its Form. (In my previous post I described how the Four Elements etc. are circumferentially bounded by Form or Spirit.)

According to Ptolemy the two lower planets VJ are "beneficial" and the two upper SM are "maleficient." Also, the two left planets SV are nocturnal and the two right MJ are diurnal. Thus as is commonly done, we can draw the Moon (C) on the left side of the Pentagram, and the Sun (O) on the right:

C	O
H	
S M	
V J	

Mercury/Hermes, as usual, stands apart from these distinctions. However, as Psychopomp (Guide of Souls), Hermes shows us the Two Shamanic Paths represented in the diagram:

```
C      O
      H
    E  F
      W A
```

The Left-hand Path, the Path of Darkness (C), takes the Way of Earth (E) down into the Watery Abyss (W). The Right-hand Path, or Path of Light (O), takes the Way of Air (A) up into the Celestial Fire (F). (There is, however, no implication here that one Way is Good and the other Evil.) Hermes (H) stands as the Master of the all.

Now let's label the corners of the Pentagon with the first letters of Greek words for the Elements:

```
U: `Udor = Water
G: Gaia = Earth
I: Idea = Form/Idea
Th: Therma = Heat    or    EI: `Eile = Sun's Warmth
A: Aer = Air
```

Depending on which abbreviation you like for Fire, you get the following Pentagrams:

```
      I      I
    G  Th    G  EI
    U A      U A
```

If you read the letters in order, starting at the Organic Transition Point, you get UGITHA or UGIEIA, the Pythagoreans' mystical name for the Pentagon. Readers of my Olympic Banishing Ritual may recall that there are two labelings, either UGIEIA = hugieia = Soundness, or the somewhat inexplicable UGITHA, with Theta replacing Epsilon-Iota, which appears at least as early as Agrippa. Though the Theta may be explained as a joined Epsilon and Iota, we see here an alternative explanation, for either is an abbreviation for the Fiery Element. Notice how the pattern of the letters, UG/I/EIA or UG/I/ThA, matches the arch structure of the Pentagon; interestingly the Greek word has a high-tone accent on the Iota (corresponding to Spirit), which seems appropriate.

By the way, if you think that there are probably enough Greek words for the Elements to get UGIEIA from many Pentagon arrangements... take my word for it; there aren't. If you don't believe me, then get yourself the big Liddell & Scott and have fun. But as far as I've been able to discover, this is the *only* arrangement that generates UGIEIA or UGITHA (let alone both) from plausible Element names (though another comes close; see below). Also I should note that the above Element names are the usual ones that appear, for example, in Aristotle, except Fire, where Pur is usual; however Empedocles (who came up with the Four Elements) uses Eelios = Sun, which is described as Thermos, for Fire (DK 31B 21; see also Kirk, Raven & Schofield 292-3). Also, though Aristotle uses Aither for the Fifth Element, Plato clearly associates it with the World of Forms (Ideai).

This brings us to an interesting property of our Pentacle (the Pentagram with Circumscribed Pentagon). In mathematical terminology the Pentagram is dual to the Pentagon (one is like the other turned inside-out). To see this draw a black Pentagram in a red Pentagon, and a red Pentagram in a black Pentagon; each Pentacle is the dual of the other. For example, if we number the first Pentagram clockwise, and transfer the numbers counterclockwise to the second Pentagram, then our Pentacles will be:

1		1	
5	2	4	3
4	3	2	5

The second Pentacle has the Even Numbers, which according to Pythagorean theory are Feminine, on the left, and the Masculine Numbers on the right; since according to the Pythagorean 1 is Androgynous (cf. Hermes Duplex), it is appropriate that it is in the middle.

Now label the first Pentacle with the Elements and Planets as described above; I will call this the Elemental Pentacle. Next trace the Elemental Pentagram in Counterclockwise Order (HVMSJ = SWFEA) and lay out these elements in Clockwise Order around the second Pentacle; they will be arranged as follows (the Sun/O and Moon/C are left in position):

C		O
	H/S	
J/A		V/W
S/E	M/F	

The resulting Dual Pentacle is interesting for a number of reasons. First observe that the Elements are arranged in the familiar Wiccan/Golden Dawn format, so henceforth I will call this the W/GD Pentacle. Because this is the dual of the Elemental Pentacle, tracing the W/GD Pentagram in Invoking Order, SFAWE, corresponds to circumscribing the Elemental Pentagon in (descending) Physical order, SFAWE. The W/GD Pentacle also displays the steps of the Alchemical Process in order; on the left we have the Lesser Work, Saturn to Jupiter and culminating in the Moon (C); on the right we have the Greater Work, Venus to Mars and culminating in the Sun (O). The entire process is governed by Mercury (Burckhardt, Alch. XV). Recall also that the Planets in the Solar side (V, M) have a Circle in their astrological symbols, while those on the Lunar side (J, S) have a Crescent; Mercury (H), of course, has both.

As is well known, Plato associates the Five Elements with the Five Platonic Solids (Tim. 55c-65c), and Kepler made a similar association between the Planets and the Platonic Solids. So let's lay out the Platonic Solids around the W/GD Pentacle; I'll abbreviate the Solids by their number of faces:

4=Tetrahedron, 6=Cube, 8=Octahedron, 12=Dodecahedron, 20=Icosahedron.

This is the result:

	12	
8		20
6	4	

Notice that the Platonic Solids fall around the circumference in increasing order of number of faces. Since the Elemental Pentagram is its dual, we know

that it can also produce the series 4, 6, 8, 12, 20 (by tracing the Pentagram).

So the W/GD Pentacle also has some interesting Correspondences, though it loses the Directional and Organic Cycles and especially the Square of Opposition, which I think is fundamental and essential.

That's all I have to say, for now, about the Elemental and W/GD Pentacles. Needless to say, they can be profound Mandalas for meditation if fully labeled with color-coded Correspondences.

* * *

I would like to point out a couple interesting properties of another Pentacle, which preserves the Square of Opposition, Directions and Organic Cycle, but loses the Metaphysical Cycle. This is the one we get if we take the view toward the Southeast (as might have been the norm among the Etruscans):

Directions Elements Planets

	*		S			H	
E	S		F	A		M	J
N	W		E	W		S	V

If we trace this Pentagram, starting at the top (Mercury/Spirit) and proceeding to the Organic Transition (Venus/Water), then we will trace the Planets in the order HVMJS (Mercury, Venus, Mars, Jupiter, Saturn) -- the so-called Chaldean Order for the Planetary Spheres, which was commonly used in ancient Greece (and is of course the modern order for the Five Minor Planets).

On the other hand, if we trace them counterclockwise around the Pentagon, then we get HMSVJ, which -- astonishingly -- is the standard Babylonian order for the Planets (Jastrow, Rel. Bab. Ass. 461n2):

Merc. (Nabu), Mars (Nergal), Sat. (Ninurta), Venus (Ishtar), Jup. (Marduk).

(Actually, the Babylonians would normally reverse this order, but that's just going around the Pentagon in the opposite direction. This also gives 5 days of the week: M/Tues, H/Wed, J/Thur, V/Fri, S/Sat) In other words, the Chaldean and Babylonian Orders are duals of each other. Therefore, I call this the Planetary Pentacle.

Finally, I'll briefly consider whether the Planetary Pentacle could generate the Pythagorean UGIEIA/UGIThA. Taking UGA as fixed we get the Pentagrams:

	Th		EI	
?	A		? A	
G	U		G U	

That is:

U: `Udor = Water

G: Gaia = Earth

I: ? = Fire

Th: Thumos = Spirit or EI = EIdos = Idea = Form

A: Aer = Air

Unfortunately there is not, so far as I know, any Greek word meaning Fire (or Heat or Sun, etc.) that begins with Iota. (BTW, Ignis is, of course, Latin, and I'm sure Pythagoras wouldn't have stooped to put a Latin Word in his formula.) Another problem with this arrangement is that, although Thumos = Spirit is reasonable enough in a psychological context, it doesn't work well in Plato's or Aristotle's metaphysical context. Finally, we lose the high-tone accentuation of Spirit in the middle of the arch structure. So the Planetary Pentacle is interesting, but ultimately packs less meaning than the Elemental Pentacle.

APPENDIX: DRAWING PENTAGRAMS AND PENTAGONS

I've paraphrased the following from Charles W. Benfield, "Constructing a Pentacle" (Nematon, Vol. I, No. 3; Beltane 1974, pp. 12, 14-16):

Procedure I

1. Draw a horizontal line, which will be the diameter of the bounding circle.
2. Bisect the line to get its center, and draw a perpendicular through this center.
3. Draw a circle around the center with the given diameter.
4. Bisect the horizontal three more times to get its sixteenth part.
5. Mark on the horizontal the point $13/16$ from its left end ($5/8$ of the right-hand radius).
6. Use the compass to measure the distance from this point to the intersection of the vertical and the circle; this is the pentagon's side.
7. With the compass set as described, set off five points around the circle's circumference.
8. Connect these points to make the pentagram.

Procedure II

1. Construct as before the circle ABCD with vertical diameter AOC and horizontal diameter DOB.
2. Bisect OD at E.
3. Set the compass to measure EA.
4. With E as the center, drop an arc AF which intersects OB at F.
5. Set the compass to measure AF; this is the side of the pentagon.
6. Step off the five points and draw the pentagram as before.

* finis *